Design of two channel mesopic photometer

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Opintopisteet (ECTS): ______________
Arvosana (1 – 5): ______________
Ohjaajan allekirjoitus: _______________________________________

TkL Pasi Manninen
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1 Introduction

Sensitivity of human eye is different at dark conditions and at bright daylight. In dark conditions, the vision is called scotopic, and in bright conditions it is photopic. Between scotopic and photopic vision is the level of mesopic vision. In those conditions, spectral sensitivity of human eye depends on brightness of light. Conventional luxmeters have fixed spectral responsivity and therefore they are not accurate to describe the illuminance in mesopic conditions.

An important application of study of mesopic vision is traffic lighting. Therefore mesopic vision was studied by CIE (International Commission on Illumination) using performance based methods. Based on these studies, the intermediate model was defined. The intermediate model included linear combination of conventional photopic and scotopic models that made it practical for implementing measurement devices.

In this work, a luxmeter for mesopic measurements was designed based on the outcome from the performance based studies. First in Chapter 2, the basics of photopic and scotopic photometry are discussed. Mesopic vision is discussed in Chapter 3 by presenting three performance based models for mesopic vision. In Chapter 4, the structure of luxmeter based on mesopic models is designed. Then the illuminance responsivity of designed meter is calculated in Chapter 5. Finally, the selected mesopic model is tested numerically in Chapter 6.

2 Scotopic and photopic photometry

Human eye has different sensitivity for wavelengths of light. For example, the sensitivity of the eye is the best to green light. To develop lighting conditions for human, we need a quantitative measure to describe the level of brightness. How can brightness be measured objectively? Sensations between humans are naturally different, even same person in same situation might give different description of brightness. Thus we need a model averaging the seeing-abilities of different people but still giving useful results. This is hard because of large adaptability of the eye at different lighting conditions.

Photometry is the science of measuring radiation the eye can use in seeing. Only power at wavelengths 360 – 830 nm is considered as visible light, while for instance infrared and ultraviolet have no use in seeing. Thus different weighting for power at different wavelengths must be applied to obtain the amount of useful light power. Weighting function defined by CIE is called as \( V(\lambda) \). [1] It is an experimentally defined function of average young human eye sensitivity to different wavelengths. Plots of this function are in Figure 1. Using the weighting, portion of visible power in spectral radiant flux \( P_v(\lambda) \) is given by

\[
F_p = K_{m,p} \int_0^\infty V(\lambda)P_v(\lambda)d\lambda, \quad K_{m,p} = 683 \text{ lm} / \text{W},
\]

which is called luminous flux and has unit lumen (lm) to notate that only visible power is included. When measuring illuminance, the amount of power itself is not an informative measure, because same power distributed in larger surface will yield less power per unit area. Thus the visible intensity is used instead. Visible intensity is defined as the luminous power falling on unity area, thus having unit lm/m² also called lux (lx) [1]. It is a prevalent standard in designing illumination. Nevertheless it is not a good measure for seeing while any level of illuminance does not guarantee sufficient visibility. For example, white wall is seen more bright than black wall, independent of how much the wall is illuminated. [2] Thus, when measuring brightness, we need to measure the light coming from the target rather than the illumination. This measure is called luminance (cd/m²) which is a useful measure for brightness of target.
Let’s take another look at the weighting function used in previous definitions. $V(\lambda)$ obtains its maximum at 555 nm wavelength. To normalize the integral (1), the values of the weighting function are scaled so that it reaches unity at that wavelength. To ease the integration further, $V(\lambda)$ is set to zero outside the visible area.

Restriction on usability of $V(\lambda)$ is that the spectral sensitivity of human eye is not constant in different illuminance levels. The peak sensitivity is at 555 nm only in bright daylight (photopic vision), but in dark conditions (scotopic vision) the peak is at 507 nm. Based on this phenomenon, another sensitivity function – called scotopic sensitivity function $V'(\lambda)$ - is also defined by CIE. Plot of this function is in Figure 1 with dashed line. Scotopic luminous flux is obtained by replacing weighting function in equation (1), which gives

$$F'_c = K_{m,s} \int_{0}^{\infty} V'(\lambda)P_e(\lambda)d\lambda, \quad K_{m,s} = 1699 \text{ lm / W.} \quad (2)$$

Luminance levels between photopic and scotopic vision are called mesopic. Mesopic vision conditions occur for example at night on normally illuminated street. The distinction between scotopic, mesopic and photopic luminance levels is not clear and therefore no exact boundary values are defined by the CIE standard. However, some rough orders of magnitude are given in Table 1. [4] The boundary in fact cannot be accurately defined, because it also depends on the eccentricity of target in the visual field. This complicates the study of mesopic photometry, but as seen in the next chapter, practical models can be defined.

<table>
<thead>
<tr>
<th>Table 1: Properties of vision at different levels of luminance. [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Luminance [cd/ m²]</strong></td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>10⁻⁶ ... 10⁻²</td>
</tr>
<tr>
<td><strong>Perception of colors</strong></td>
</tr>
<tr>
<td><strong>Operative cells</strong></td>
</tr>
<tr>
<td><strong>Luminosity function</strong></td>
</tr>
<tr>
<td><strong>Sensitivity peak [nm]</strong></td>
</tr>
</tbody>
</table>
3 Standardization of mesopic photometry

3.1 Preliminaries

In the night time, a driver has to work in wide range of luminance levels. But the actual luminance alone is not sufficient to measure the suitability of lighting. Thus in research of vision, many factors must be taken into account. The driver has to detect and recognize objects, perceive movements and relations between objects. These cases are also common in other conditions than driving, so researching those benefit other purposes like designing safety lighting too. But how can performance be measured? By the definition of International Lighting Vocabulary, visual performance is measured e.g. by the speed and accuracy of the performance.

Some factors must be taken into account when defining the mesopic sensitivity function. This is because the sensitivity of eye depends on many conditions.

Firstly, the sensitivity is different in different angles of eccentricity. This is because fovea in the retina has mostly cones, while rods are in minority there. Therefore foveal vision, which extends to 2° from center of view, has different sensitivity than the rest of the retina, where both rod and cone cells are present. In fact, experiments with foveal vision have proved that photopic luminosity function is valid also at mesopic levels, just because of lack of rods in fovea. Thus mesopic luminosity function is needed only for peripheral vision. Because of this dependency, viewing angle must be standardized with mesopic measurements. For example, eccentricity of 10° was used in experiments of MOVE (Chapter 3.2.1).

Secondly, the size of visual target has an effect on the performance. Therefore, the visual size of target must be standardized. For example, a target size of 2° has been used in the studies.

Third, the spectral properties of the light source must be characterized. Scotopic-to-photopic luminous flux ratio (S/P-ratio) is used for this purpose. The S/P-ratio is defined as a ratio of scotopic and photopic luminous flux of a light source. [6] Although this ratio neglects information of the actual power spectral density, it gives a simple measure of the spectrum.

In the next, three models using performance based study of mesopic vision are introduced.

3.2 Models for mesopic vision

3.2.1 MOVE

Multi-disciplinary MOVE (Mesopic Optimization of Visual Efficiency) consortium has researched mesopic vision with methods of visual performance. Performance was studied in three phases: 1) whether the object was detected, 2) how quickly it was detected and 3) whether the object was recognized. Detecting an object was tested by defining achromatic threshold for contrast. Achromatic threshold was defined as the smallest difference in achromatic contrast that could be distinguished from the background. The difference in achromatic contrast was defined as the relative difference in luminance between target and background. This is an important measure, because it describes how accurately the driver can detect road and obstacles to make the driving safe. The speed of detection was measured as the time between presentation of object and testee’s reaction to it. Further, recognition of object was measured with exercises measuring the detection of details in the object.

Chromatic response of the eye is not linear; there are three peaks in the sensitivity, caused by three different types of cone cells in retina. This was taken into account in the chromatic MOVE-model, which was not linear. Anyway, a linear model
Figure 2: Mesopic sensitivity function at different values of parameter \( x \). When \( x=0 \), the sensitivity function is the same as \( V'(\lambda) \cdot K_{m,s} \) (the highest black curve in the figure) and when \( x=1 \), the function is the same as \( V(\lambda) \cdot K_{m,p} \) (the green curve).

\[
V(\lambda)_{\text{MOVE}} = \frac{1}{M(x)} \left[ xV(\lambda) + (1 - x)V'(\lambda) \right],
\]

was found to be more practical than a non-linear. As seen in the Figure 2, the parameter \( x \) is used to gradually move the shape of mesopic luminosity function \( V(\lambda)_{\text{MOVE}} \) between conventional scotopic \( V'(\lambda) \) and photopic \( V(\lambda) \) functions. \( M(x) \) is to normalize maximum of \( V(\lambda)_{\text{MOVE}} \) to unity.

The three different visual performance tasks were modeled separately, but finally single model was found to suit for all cases. Mesopic luminance in the MOVE linear model (10° eccentricity) was defined with an iterative formula

\[
x_{n+1} = a + b \log_{10} \left( \frac{1}{M(x_n)} \left( x_n \frac{L_p}{K_{m,p}} + (1 - x_n) \frac{L_s}{K_{m,s}} \right) \right), \quad 0 \leq x_{n+1} \leq 1
\]

\[
M(x_n) = \max \{x_n V(\lambda) + (1 - x_n)V'(\lambda)\} \approx 1 - 0.65x_n + 0.65x_n^2,
\]

where

\[
a = 1.49, \\
b = 0.282, \\
K_{m,p} = 683 \text{ lm/W and} \\
K_{m,s} = 1699 \text{ lm/W}.
\]

Finally, after wanted number of iterations is performed, the value of \( x \) is inserted in the formula

\[
L_{\text{mes}} = \frac{xL_p + (1-x)L_sV'(\lambda_0)}{x + (1-x)V'(\lambda_0)} \Rightarrow V'(\lambda_0) = 683/1699.
\]
The value of $x$ was noticed to be close to unity at 10 cd/ m$^2$ (measured photopically), as seen in Figure 3. At that level, the S/P ratio had negligible effect on $x$, meaning that photopic luminosity function was suitable for explaining the performance. Instead, at 1 cd/ m$^2$ (parameter $x \approx 0.7$) significance of S/P-ratio begun to increase. Furthermore the lower the luminance, the more the S/P-ratio changed parameter $x$. Value $x \approx 0.1$ was obtained at 0.01 cd/ m$^2$, meaning that mostly scotopic function could be used in those levels of luminance. [6]

![Figure 3: Some values of parameter x in MOVE-model at photopically measured luminance levels 0.01 – 10 cd/ m$^2$. Spectral distribution of light source was characterized with S/P-ratio. [6].]
3.2.3 The intermediate model

The intermediate model was based on results from MOVE and X-model. Most difficult problem in combining the two models lies in differences in results of chromatic tests: As the X-model tested both chromatic and achromatic performance, MOVE counted mostly on the chromatic measurements. Comparing the results revealed that difference between the models is largest at lighting of narrow wavelength band, while at broad-band and almost-white lighting conditions the differences are smaller. In real life conditions, multi-wavelength lighting is eventually most common. Therefore MOVE and X-model can be combined.

MOVE defined the mesopic-photopic transition to occur at 10 cd/m². Despite that, the X-model defined this point to be much lower, at 0.6 cd/m². The intermediate model locates the point somewhere between 3-5 cd/m². The difference between points of MOVE and X-model was explained with nature of studies: the X-model measured only one kind of performance leading to limited view, and the MOVE measured wide range of performance leading to inaccurate and complicated results.

Two limits for mesopic-photopic transition point were tested in the intermediate model. The MES1 model defined the mesopic area between 0.01 cd/m² and 3 cd/m², and the MES2 model between 0.005 cd/m² and 5 cd/m².

The MES1 model is a linear combination of the photopic and scotopic luminosity functions. The model gives the mesopic luminosity function

\[ V_{mes}(\lambda) = m_1 V(\lambda) + (1 - m_1) V'(\lambda), \]  

\[ m_1 = \begin{cases} 
1, & L_{mes} \geq 3.0 \text{ cd/m}^2 \\
0, & L_{mes} \leq 0.01 \text{ cd/m}^2 \\
0.404 \log L_{mes} + 0.807, & \text{otherwise} 
\end{cases} \]  

An iterative procedure must be applied to solve this, because the \( L_{mes} \) is dependent on the \( V_{mes} \).

The MES2 model is

\[ M(x)V_{mes}(\lambda) = m_2 V(\lambda) + (1 - m_2) V'(\lambda), \]  

where \( M(x) \) is a normalization function to scale the maximum value to unity. Parameter \( m_2 \) is solved iteratively with

\[ m_{2,0} = 0.5, \]  

\[ L_{mes,n} = \frac{m_{2,(n-1)}L_p + (1 - m_{2,(n-1)})L_s V'(\lambda_0)}{m_{2,(n-1)} + (1 - m_{2,(n-1)})V'(\lambda_0)}, \]  

\[ V'(\lambda_0) = \frac{683}{1695}, \]  

\[ m_{2,n} = \begin{cases} 
1, & L_{mes} \geq 5.0 \text{ cd/m}^2 \\
0, & L_{mes} \leq 0.005 \text{ cd/m}^2 \\
0.7670 + 0.3334 \times \log L_{mes,n}, & \text{otherwise} 
\end{cases} \]  

In the equations, \( n \) is the iteration index. [4] Equations (11) – (13) are possible to use in two channel photometer, which measures the photopic and scotopic luminances \( L_p \) and \( L_s \) respectively and then iterates the parameter \( m_2 \) to obtain the \( L_{mes} \) accurate enough.
4 Design of mesopic photometer

4.1 Operating principle and requirements

To measure mesopic illuminance, several alternatives exist. These include for example:

1. Measuring the spectral power distribution of light with spectroradiometer and integrating numerically the spectrum with weighting of mesopic luminosity function.
2. Spectrally adjustable filter consisting of several layers of adjustable filter disks, each layer filtering particular band of spectrum so that mesopic responsivity is obtained approximately.
3. Two channel broad-band measurement: scotopic and photopic illuminances are measured and mesopic illuminance is calculated from these.

Chromatic MOVE-model could be applied with Alternative 1. Method is accurate, but in the measurement instrument, the light is dispersed into wavelength components making it hard to measure low lighting levels. Moreover, spectrometer measurement would be impractical and expensive. Alternative 2 is also expensive, and complicated because the control system for the filter adjustment is needed.

Alternative 3 is the most practical way to realize, since only two illuminance meters with fixed filters are needed. In addition, couple of linear models for mesopic illuminance exists as seen in the Chapter 3. Alternatives 1 and 2 are not considered in this document, thus the target is to design a two channel photometer.

Detector must be able to measure low mesopic and scotopic light levels as well as photopic levels accurately. Therefore the measuring range should extend from 0.01 cd/m² to 10 cd/m². Because scotopic lighting levels are very low, sensitivity of the detector must be maximized and losses in optics must be minimized.

Measuring photopic and scotopic illuminance simultaneously must be enabled. Thus own detectors for photopic and scotopic measurements are required. Because slightly heterogeneous fields must be possible to measure with the photometer, feeding equal light into the two channels must be ensured.

Light coming from different directions must follow the Lambert’s cosine law to realize the measurement of intensity. Diffuser is used to mimic the cosine law.

A two channel optical flux density meter has been developed [7]. That meter can be modified so that photopic and scotopic illuminance is measured and mesopic illuminance is calculated in real time.

4.2 Structure

In this chapter, the alternative realizations of a two channel photometer are discussed.

Because of simultaneous measurements with two channels, light fed into each channel must be equal. Beam splitter may be used to split diffuser light into two similar fields. Unfortunately the beam splitter increases the distance between the aperture and photodetector leading to lower sensitivity. Beam splitter might also distort the spectrum. Another way is to use fully separate measurement devices with own aperture for each channel, as in Figure 5 a). Problem is then to minimize the distance d between two channels so that heterogeneous field does not cause difference between channels. One way to shorten the distance is to place both detectors under common diffuser as shown in Figure 5 b).
Figure 4: Structure of photometer with photopic and scotopic channels.

Figure 5: Some alternative designs to exclude the beam splitter: a) fully separate channels and b) scotopic and photopic channels using the same diffuser. The distance between center axes of photodetectors is \( d \).

4.2.1 Entrance optics

For illuminance measurement, a diffuser must be added before detectors as in Figure 4. Diffuser material is usually Teflon. Purpose of the diffuser is to smooth the light homogeneous so that the receiver complies with the Lambert’s cosine law. Some illuminance meters have diffusers with convex or hemi spherical profile, while others have flat ones. Shaped diffuser is better in sense of the Lambert’s cosine law [8], but will cause problems in defining the distance between target and diffuser. This vagueness in distance complicates calibration of the system and therefore shaped diffusers are rejected herein.

The material of diffuser absorbs light. Therefore the diffuser must be as thin as possible to minimize the losses. Diffuser also scatters light. In laboratory conditions the solid angle of the illuminance measurement is narrow and thus the diffuser can be removed to increase the sensitivity. In field conditions the diffuser is necessary.

The area of aperture must be clearly defined, because the response of meter is dependent on it. Different sequences of diffuser and aperture might have different properties. For instance, the diffuser can be installed before or after the aperture or even in the aperture itself. If the diffuser was after the aperture,
the area would be clearly defined by the aperture. Nevertheless the cosine law characteristics would suffer because of the distance between the front surface of aperture and diffuser. If the diffuser was located before the aperture, the cosine law would possibly come true more accurately.

Light intensity after diffuser decreases along with increasing distance. Therefore photodiodes must be located as close to the diffuser as possible to maximize the sensitivity of equipment. The aperture will usually be that small (diameter < 10 mm) that in many cases the achieved homogeneity is smaller problem than the sensitivity. Thus the target is to get the photodiode as close to the diffuser as possible. Gigahertz Optik manufactures 20 mm thick illuminance meters whose thickness complies with ANSI Lumen standard [9]. This low thickness is not target here, but gives some advice in design.

A good way to increase sensitivity is to increase the area of aperture. The size of aperture is limited by the size of active area of photodetector, and thus maximizing the size of active area will improve the sensitivity.

4.2.2 Photodetector

Sensitivity of photomultiplier tube (PMT) can be more than 100,000 times the sensitivity of conventional photodiodes. Moreover, they also provide very high signal-to-noise ratio (SNR). [10]. Respectively, they are more expensive, big-sized and need very high voltages [11], which decline their usability in portable equipment. However, some photomultipliers have spectral limitations, which are useful in photopic and scotopic measurements. For example, photomultiplier sensitive at wavelengths 185 – 850 nm exists [12]. Although PMT cannot be used as portable device, it might be good choice for calibration activities in laboratory conditions.

Avalanche photodiodes (APD) are also more sensitive than conventional photodiodes. They are lower cost and smaller sized than photomultipliers and they also need much lower operating voltage. For example, OSD3-E in Table 2 requires reverse bias less than 15 V. However, worse SNR is a problem. [13] To increase sensitivity and SNR, hundredths of APRs can be coupled in parallel as a matrix. This structure is called silicon photomultiplier (SiPM). It can be used even in photon counting. However, special equipment is required to read the SiPM chip. A drawback is that quite large voltage is needed to supply the detector. [14]

Dark current produced by the photodetector must also be taken into account, because it decreases the SNR. Dark current is also temperature dependent, which will reduce accuracy especially at low signal levels.
### Table 2: Some commercial photodiodes and their characteristics.

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Manufacturer</th>
<th>Detector</th>
<th>Peak [nm]</th>
<th>Sensitivity [A/W]</th>
<th>I₀ [pA]</th>
<th>Price [€]</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 a)</td>
<td>Hamamatsu</td>
<td>S1133</td>
<td>560</td>
<td>0.3 0.27 0.3</td>
<td>10</td>
<td>6.51</td>
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<td>5 a)</td>
<td>Hamamatsu</td>
<td>S1087</td>
<td>560</td>
<td>0.3 0.27 0.3</td>
<td>10</td>
<td>5.83</td>
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<tr>
<td>-</td>
<td>Hamamatsu</td>
<td>S1223</td>
<td>960</td>
<td>0.6 0.31 0.36</td>
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<td>-</td>
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<td>Hamamatsu</td>
<td>S5973-02</td>
<td>760</td>
<td>0.45 0.4 0.42</td>
<td>1</td>
<td></td>
<td>APD, Vᵦ&lt;20V</td>
</tr>
<tr>
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<td>S5973-01</td>
<td>760</td>
<td>0.52 0.25 0.31</td>
<td>1</td>
<td>17.66</td>
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<td>5 c)</td>
<td>Vishay</td>
<td>TEMD6010FX01</td>
<td>540</td>
<td>1.19⁻  -  -</td>
<td>-</td>
<td>2000</td>
<td>GaAsP</td>
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<td>GaAsP</td>
</tr>
<tr>
<td>5 e)</td>
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<td>S10362-11</td>
<td>400</td>
<td>-   -    -</td>
<td>-</td>
<td>-</td>
<td>SiPM, 70V</td>
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<tr>
<td>6</td>
<td>OSI</td>
<td>PIN-10AP*</td>
<td>&quot;V(λ)&quot; 0.27</td>
<td>-   -    -</td>
<td>-</td>
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<td></td>
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<tr>
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<td>OSD3-E*</td>
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<td>77346</td>
<td>420</td>
<td>34 000 26 000 19 000</td>
<td>- 956.00</td>
<td>PMT, 1000V</td>
<td></td>
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</tbody>
</table>

*Includes fixed V(λ) filter

⁻These are calculated from 0.4 nA/lx given in the TEMD6010FX01 datasheet or 6.6 nA/lx given in the OSD3-E datasheet. At 555 nm wavelength 1 W/lx = A/[KᵦV(555nm)] where active area A=0.23 mm² for TEMD6010FX01 and A = 3 mm² for OSD3-E.

PIN avalanche photodiodes are fast, but they have low SNR. Because speed is not important herein, averaging filters may be used to improve the SNR. S5973-02 is a PIN photodiode. As seen in Figure 6 b), its spectral response has flat maximum, and the sensitivity is high at short wavelengths. Especially sensitivity for the scotopic peak wavelength is the highest compared to the other photodiodes as seen in Table 2. Therefore it is profitable for the scotopic channel.

Photodiode-filter combination is a compact alternative for separate photodetector and filter. For example PIN-10AP is a silicon photodiode with filter making its response to match with CIE photopic luminosity function. The peak sensitivity is 0.27 A/W as seen in Table 2. This is lower than the sensitivities of other photodiodes, but we shall observe that the filter needed for those has transmittance maximum always less than unity. For example S1223 has sensitivity 0.36 A/W at 555 nm. To compare this to the PIN-10AP we see that if we must use filter with transmittance maximum lower than (0.27 A/W) / (0.36 A/W) = 0.75, then the PIN-10AP is still more sensitive than S1223 with its filter.

Some GaAsP photodiodes have spectral sensitivity limited into visible area. For example TEMD6010FX01 has spectral sensitivity which takes after the photopic luminosity curve. However, this model has very small active area and it is not sensitive enough to provide accurate results. Thus no GaAsP photodiode were found suitable.
a) S1133, S1087 [15]

b) S5973 [15]

c) TEMD6010FX01 GaAsP photodiodes and the photopic curve. [16]

d) Sensitivity of GaAsP photodiodes. The scotopic curve was added to the picture afterwards. [15]

e) S10362 quantum efficiency [15]

f) Responsivities of some Oriel photomultiplier tubes [11]

Figure 6: Spectral characteristics of different photodetectors. The horizontal axes are wavelengths in nanometers.
Figure 7: PIN10-AP response and photopic luminosity function. Photodiode has fixed photopic filter which approximates CIE curve with 4% error area between the curves. [17]

4.2.3 Filter

Custom fitted filters are needed mimicking the CIE sensitivity functions. Custom filters can be constructed from standard filters available in stocks of colored glass manufacturers. To make a custom filter, standard filters with suitable spectral transmittance properties are first made in right thickness to obtain the wanted transmittance. Finally the filters are cemented together. Bandpass filter composition is possible to construct from lowpass and highpass filters. For example UQG Optics manufactures custom filters with that technique. However, the two filters must be selected from standard filters and thus the spectral response might become imperfect. Therefore searching suitable filters for making accurate response is not included in this document.

Complete photopic filters are also available in stocks [18]. Problem with those is that their spectral response is not fitted for all photodiodes as seen in Figure 12. Although commercial photopic filters could be adjusted with additional filters so that the total spectral response complies with the wanted one, the transmittance maximum would decrease. Commercial photopic filters are, however, easiest way to get approximate filter match.

Custom filters might also be manufactured by adding absorbing compounds into glass or plastic. Since these filters consist only of one layer, surface reflections would be minimized. However, no suppliers with this manufacturing technique were found.

Surface of a filter is an interface between air and glass (or plastic). Because of the difference between refractive indices of these two materials, reflection occurs. This reflection for light beam perpendicular to the surface is about 4 % for each interface [20]. Coating the surface reduces the loss, for example UQG Optics’ coating service can reduce reflection to less than 1.5 % [19]. Unfortunately this enhancement is dependent on wavelength [20]. Transmission specifications of coating must be taken into account when
determining filter transmittance. Thickness of coating should be selected to give maximum in transmittance where transmittance of photopic (or scotopic) sensitivity function has maximum. Taking advance of this extra filter could also include fine-tuning the spectral response of the main filter, but this is out of the scope herein.

To make the relative spectral sensitivity of photodetector $R_{rel}(\lambda)$ equal to the $V(\lambda)$, the required transmittance function of filter must equal to

$$
\tau(\lambda) = \frac{V(\lambda)}{S_{rel}(\lambda)},
$$

(14)

where $S_{rel}(\lambda)$ is the normalized sensitivity of photodiode. For scotopic filter the $V(\lambda)$ is replaced with the $V'(\lambda)$. In Figure 8, spectral response of required photopic filter is defined for S1087 photodiode. Scotopic filter is defined in Figure 9. Because the sensitivity of photodiode is relatively flat compared to the CIE curves, the required filter has very similar curve as the CIE curves have. This is seen even better in Figures Figure 10 and Figure 11, which define the required filters for S5973-02 photodiode.

Figure 8: Transmittance curve of required photopic filter. The photo sensitivity of photodiode S1087 in Figure 6 a) was normalized to give values between 0 and 1. Finally the filter needed to realize photopic $V(\lambda)$ was defined using (14). The spectral sensitivity of photodiode is of the same shape as the $V(\lambda)$ function, thus requiring less filtering and giving more efficiency. *
Figure 9: Transmittance curve of required scotopic filter. In the same way as in Figure 8, the filter needed for scotopic channel - having S1087 as detector - was defined using (14) but scotopic $V'(\lambda)$ was used in the equation instead of photopic luminosity.*

Figure 10: Transmittance curve of required photopic filter. The photo sensitivity of PIN photodiode S5973-02 in Figure 6 d) was normalized to give values between 0 and 1. Finally the filter needed to realize photopic $V(\lambda)$ was defined. As the spectral sensitivity of PIN photodiode is quite flat, filter is substantially the same as definition of $V(\lambda)$.*

Figure 11: Scotopic filter needed for scotopic channel with S5973-02.*
Figure 12: Comparison of the fixed photopic filter FPE-1250 and the required filter. FPE-1250 does not match the wanted filters. The transmission maximum of filter is at 536.4 nm, which is too short a wavelength. [18] It is not a good filter for scotopic channel neither.*
*Because the numeric data for plotting the photodiode sensitivity and the filter transmittance was not available, the values were read manually from the plots in datasheet of component and the curves were re-plotted.

Because transmittance curve of required filter resembles the standard photopic luminosity curve, a photopic filter would be used. FPE-1250 is an example of commercial photopic filter. Comparison of this filter and our target filters is in Figure 12. Although FPE-1250 is specified to photopic filter, it seems to give bad match with our target. Some other bandpass filters are presented in Table 3. If manufacturing a custom filter is not possible, then 10BPF70-500 could be used as scotopic filter although it has sharper pass band than CIE definition requires.

Table 3: Properties of different commercial filters.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Diameter [mm]</th>
<th>Peak [nm]</th>
<th>Max transmittance</th>
<th>Criticism</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPE-1250</td>
<td>12.5</td>
<td>536.4</td>
<td>0.65</td>
<td>Peak transmittance at wrong wavelength</td>
<td>£39 [18]</td>
</tr>
<tr>
<td>FSR-BG18</td>
<td>12.7</td>
<td>510</td>
<td>0.83</td>
<td>Half-band-width 250nm -&gt; too broad</td>
<td>57 € [21]</td>
</tr>
<tr>
<td>10BPF70-500</td>
<td>25.4</td>
<td>500</td>
<td>0.6</td>
<td>Half-band-width 70nm -&gt; too sharp</td>
<td>135 € [22]</td>
</tr>
</tbody>
</table>
4.2.4 Estimating the measuring current levels

Very low illuminance levels have very low power. Take an example of scotopic luminance \( L = 0.01 \text{ cd/m}^2 \) detected through a \( D = 8 \text{ mm} \) diffuser. Let the wavelength of light be 507 nm. If the transmittance of the input optics is \( \tau = 0.005 \), then the power going through the diffuser is

\[
P = \tau \left( \frac{\pi \text{lm}/\pi}{K_{m,s}} \right) * \pi * \left( \frac{D}{2} \right)^2 = 0.005 * \frac{\pi \text{lm} / \text{cd} + 0.01 \text{ cd/m}^2}{1699 \text{ lm/W}} * \pi * \left( 0.008 \text{ m/2} \right)^2 \approx 4.6 \text{ pW} . \tag{15}
\]

Let the photodiode sensitivity be \( S = 0.3 \text{ A/W} \), which is a typical sensitivity for photodiodes in Table 1. Thus the photocurrent obtained is

\[
J = P * S \approx 1.4 \text{ pA}. \tag{16}
\]

When the diffuser is removed (\( \tau = 1 \)), \( P \approx 920 \text{ pW} \) and \( J = 280 \text{ pA} \). If the power is not monochromatically concentrated to 507 nm the luminosity function will reduce the sensitivity more. Furthermore, the internal shunt resistance of photodiode will reduce the current output. Problems arise when this low current must be amplified. At least \( 10^3 \) amplification is needed to amplify 1 pA to desirable mA-level. The amplifying is discussed more in the next section.

4.3 Measurement circuit

Since current produced by photodiode is of the magnitude of 1 pA, very much care must be taken of current-to-voltage conversion. Amplifier should be able to give amplification of \( 10^3 \) or more, with good signal to noise ratio and linearity. Conventionally a transimpedance amplifier (Figure 13) driven by operational amplifier is used for this. However, non-idealities of operational amplifier cause harm with presence of high amplification. Problems include finite input resistance, finite open-loop gain, offset current, offset voltage and temperature dependency of offset current and voltage. Moreover, the transimpedance amplifier must have expensive, big-sized and thus damage-prone precision resistors. One discrete resistor is needed per each amplification level.

Because of restrictions with transimpedance amplifier, an alternative design of amplifier is considered. Switched integrator amplifier (SIA) seems to be plausible an amplifier for accurate optical measurements. As in Figure 14, the amplifier is based on capacitor integrating the current to be measured. After integration is complete, an operational amplifier is used to measure the voltage in the capacitor. Some logic is also needed to control the switches so that integrating cycles are repeated.

SIA has many advantages superior to transimpedance amplifier. By extending the integration time, larger amplification is obtained. This makes it easy to control the amplification. In practice, current/voltage gains up to \( 10^{12} \) have been reached and linearity of 0.0011 % has been reported at gain \( 10^8 \). Since the SIA ideally has neither shunt nor feedback resistor, no thermal noise (Johnson noise) is generated by the amplifier circuit. Instead, shot noise in semiconductors and variation in integration time are the main sources of noise. Accurate measurement of time can despite be easier to obtain than accurate resistances, and if the clock from signal processor is utilized, no extra precision components are needed. [23]

A switched capacitor (SC) current-to-voltage converter is presented in Figure 15. It has two similar feedback circuits. While one circuit is charging, the other is discharging. The output voltage \( V_{out} \) is read after integration is complete, and then the roles of the two circuits are swapped.

Reasonable linearity and amplification rates are estimated to be possible to achieve with SC current-to-voltage converter. It is also possible to control the amplification by altering the switching frequency. [24,25]
Figure 13: A transimpedance amplifier with feedback resistor $R_f$. $I_{in}$ is the current to be measured and $V_{out}$ is the output voltage of amplifier.

Figure 14: A basic schematic illustrating the operation principle of SIA. During the integration, capacitor C is charging via “Hold” switch. To start new integration period, “Reset” is used to discharge the capacitor.

Figure 15: Principle of SC current-to-voltage converter. In phase 1, switches A and B are in position 1 and capacitor C2 is discharged via switches r2. In phase 2, switches A and B are in position 2 and capacitor C1 is discharged via switches r1.
5 Calculating the mesopic illuminance from photometer outputs

In this chapter we calculate the illuminance responsivity to convert the photodiode current into illuminance. This is done by assuming the current linearly dependent on illuminance \( E \). Therefore, with known photopic illuminance \( E_0 \) we get photodiode current

\[
J_{0,p} = k_p E_{0,p},
\]

(17)

where \( k_p \) is the illuminance responsivity of photopic channel. Luxmeter output current \( J_{0,p} \) is determined by the spectral responsivity

\[
R_p(\lambda) = \tau_{d,p} \tau_p(\lambda) S_p(\lambda)
\]

(18)

where \( \tau_{d,p} \) is the transmittance of input optics, \( \tau_p(\lambda) \) is the filter transmittance and \( S_p(\lambda) \) is the spectral responsivity of the photodiode. By defining relative spectral responsivity \( R_{rel,p}(\lambda) = R_p(\lambda)/R_p(555\text{nm}) \) we can write the output current

\[
J_{0,p} = A_p R_p(555\text{nm}) \int R_{rel,p}(\lambda) P_e(\lambda) d\lambda
\]

(19)

where \( A_p \) is the active area of the photodiode.

The illuminance is defined by

\[
E_{0,p} = K_{m,p} \int V(\lambda) P_e(\lambda) d\lambda.
\]

(20)

Now the illuminance responsivity in (17) can be solved

\[
k_p = \frac{J_{0,p}}{E_{0,p}} = \frac{A_p R_p(555\text{nm})}{K_{m,p}} \frac{\int R_{rel,p}(\lambda) P_e(\lambda) d\lambda}{\int V(\lambda) P_e(\lambda) d\lambda} = \frac{A_p R_p(555\text{nm})}{K_{m,p}} \frac{1}{C_p},
\]

(21)

where we have used the color correction

\[
C_p = \frac{\int V(\lambda) P_e(\lambda) d\lambda}{\int R_{rel,p}(\lambda) P_e(\lambda) d\lambda}.
\]

(22)

Illuminance responsivity \( k_p \) can now be used to convert the photodiode current into illuminance, but conversion is accurate only when the power spectral distribution \( P_e(\lambda) \) is the same as with defining the color correction.

Similarly, for scotopic channel we can solve

\[
k_s = \frac{J_{0,s}}{E_{0,s}} = \frac{A_s R_s(507\text{nm})}{K_{m,s}} \frac{\int R_{rel,s}(\lambda) P_e(\lambda) d\lambda}{\int V(\lambda) P_e(\lambda) d\lambda} = \frac{A_s R_s(507\text{nm})}{K_{m,s}} \frac{1}{C_s},
\]

(23)

where \( A_s \) is the active area of the photodiode, \( R_{rel,s}(\lambda) = R_s(\lambda)/R_s(507\text{nm}) \) is the relative spectral responsivity of the scotopic channel, \( R_s(\lambda) = \tau_{d,s} \tau_s(\lambda) S_s(\lambda) \) is the spectral responsivity of the scotopic channel consisting of transmittance of input optics \( \tau_{d,s} \), scotopic filter transmittance \( \tau_s(\lambda) \) and photodiode spectral responsivity \( S_s(\lambda) \), \( R_s(507\text{nm}) \) is the responsivity of the photodiode at 507 nm and color correction is

\[
C_s = \frac{\int V(\lambda) P_e(\lambda) d\lambda}{\int R_{rel,s}(\lambda) P_e(\lambda) d\lambda}.
\]

(24)
For intermediate model discussed in Chapter 2.2.3 we need photopic and scotopic luminances, but in the previous, illuminance was calculated. To convert illuminance into luminance, we use

\[ E = aL, \quad a = \pi \text{ steradian} \left( \frac{\text{lm}}{\text{cd}} \right) \]  

(25)

which is eligible if aperture of meter is Lambertian with perfect reflectance [26]. Thus the final formula for calculation of mesopic luminance is obtained by inserting mesopic and scotopic luminances

\[ L_p = \frac{k_p}{a} J_p = \frac{K_{m,p}}{a A_p R_p(555\text{nm})} C_p J_p \]  

(26)

and

\[ L_s = \frac{k_s}{a} J_s = \frac{K_{m,s}}{a A_s R_s(507\text{nm})} C_s J_s \]  

(27)

in equation (12) and iterating the value for \( L_{mes} \). This mesopic luminance can be converted back to the mesopic illuminance by equation (25).
6 Testing the calculation

The MES2 model was implemented with MATLAB (see Appendix) to test where it converges. Different combinations of photopic and scotopic luminances between 0.01 cd/m$^2$ and 30 cd/m$^2$ were fed into the model and number of iterations $n$ needed to convergence of $|m_{z,n} - m_{z,(n-1)}| < 0.0001$ was calculated. The results are in Figure 15. In testing the calculation, the convergence was reached usually with 10-20 iteration times. When the photopic luminance reached the limit 5 cd/m$^2$, the number of iterations needed for convergence increased steeply. Similar effect was seen also with increasing scotopic luminance over 10 cd/m$^2$. Therefore the program calculating the mesopic luminance should check that the luminances are below those upper limits.

The model was noticed to have different sensitivity for inaccuracies in the photopic and scotopic luminances in different luminance levels. As seen in the Figure 16, especially increasing of the photopic luminance will make the model more sensitive to errors in luminance measurement. Consequently, the mesopic luminance calculated by MES2 model is more insensitive for filter errors at low luminance levels.

![Figure 15: Convergence of MES2 model with different combinations of photopic and mesopic luminance. The red areas need iteration at least 1000 times, as white areas need less than 1000 times. When photopic and scotopic luminances are big enough, it was noticed that the model does not converge at all. Nevertheless the model seems to converge if the photopic luminance is below 5 cd/m$^2$ and scotopic luminance is below 10 cd/m$^2$.](image-url)
Figure 16: Absolute value of gradient of mesopic luminance in MES2 model. The more hot color, the more sensitive the model is for inaccuracies in the photopic and scotopic channels. There seems to be some numerical problems when going under 0.005 \text{cd/m}^2 in photopic luminance.

7 Conclusions and future work

In this document, the analysis of mesopic lighting levels was discussed. The intermediate model based on MOVE linear model and the x-model was selected for further consideration. The X-model is a function of photopic and scotopic luminances and therefore it can be used in two channel broad-band measurement.

For measuring photopic and scotopic illuminances, a two channel illuminance meter was designed. Because low lighting levels must be measured, maximizing the responsivity of meter became the main interest in designing. In designing it turned out that PMT without diffuser would be the best in calibration activities in laboratory conditions, because high gain and sensitivity would be achieved. For use as portable device, diffuser must be used and PMT must be substituted with sensitive avalanche photodiode. The current output of photodetector must be amplified before AD-conversion. SC and SIA current-to-voltage converters were considered to be suitable way to amplify the very low currents to be measured.

The designed luxmeter measures photopic and scotopic illuminance. The measured illuminances were designed to be fed into a computer program to calculate the mesopic illuminance according to a MES2 model. The MES2 model was tested with different inputs, and it was found to work well.

In the future, the mesopic photometer can be built by manufacturing photopic and scotopic filtered radiometers. Two photodetectors are required and SC/SIA current-to-voltage converters for them. Also a two channel AD-converter is needed. The computer program including the MES2 model might be implemented with Labview. The two channel photometer could be used for calibration activities or even for the measurements in the application field.
8 References

Appendix

MATLAB function for calculating mesopic luminance

```matlab
function [L_mesopic, n_iter] = x_model(L_p, L_s)
% Calculates the mesopic luminance from photopic and scotopic luminances L_p and L_s respectively. Calculations according to MES2 model.

m = 0.5;
V_0 = 683/1699;
k_max = 1000; % maximum number of iteration
delta = 0.0001; % stop criteria
L_mes = zeros(k_max, 1);
for k = 1:k_max
    L_mes(k,1) = (m*L_p + (1-m)*L_s*V_0) / (m + (1-m)*V_0);
    m_n = real(0.767 + 0.3334*log10(L_mes(k,1)));
    if (m > 1)
        m_n = 1;
    end
    if (m < 0)
        m_n = 0;
    end
    if (abs(m - m_n) < delta)
        break
    end
    m = m_n;
end
L_m = L_mes(find(L_mes),1);
n_iter = size(L_m,1);
L_mesopic = L_m(end,1);
```

MATLAB script for testing the convergence and gradient

```matlab
N=100;
L = logspace(-2,1.5,N);
L_mes = zeros(N,N);
R = zeros(N,N);
for k=1:N;
    for j=1:N;
        [L, n_iter] = x_model(L(1,k), L(1,j));
        L_mes = zeros(N,N);
        R(k,j) = n_iter;
    end
end

figure(1)
surf(log10(L),log10(L),R)

figure(2)
surf(log10(Lp),log10(Ls),log(abs(gradient(L_mes))))
colorbar
xlabel('log_10(L_p)', 'fontsize',16);
ylabel('log_10(L_s)', 'fontsize',16);
```